The wing of triangular planform is one of the basic elements of aircraft and, therefore, the flow and aerodynamic heating are matters of increasing interest. It was shown in [1-7] that on the upper surface of the wing there may exist regions of intense heat transfer, the so-called heat flux peaks. The formation of peaks is linked to the appearance of separated vortex flows on the leeward wing surface.

The values of the peaks and their locations depend on the Mach number $M_{\infty}$, the Reynolds number Re, the angle of attack $\alpha$, and the wing configuration. For $M_{\infty}=5$ and $\operatorname{Re}_{1}=10^{7} 1 / \mathrm{m}$ (see, e.g., [3]), they are largest for $\alpha \approx 5^{\circ}$ and are a maximum for a sweepback angle $x=$ $75^{\circ}$ in the range investigated experimentally. Here the maximum values of Stanton number $S t_{\infty}$ in the peaks is several times larger than the corresponding values determined for $\alpha=$ 0 . For $M_{\infty}=10.3, \operatorname{Re}_{1}=2.2 \cdot 10^{6} 1 / \mathrm{m}$, and $\alpha=5^{\circ}$, no peaks of $S t_{\infty}$ are observed [6] on the leeward wing surface with $\chi=80^{\circ}$. The local heat-transfer maximum then appears near the plane of symmetry of the wing at larger $\alpha$.

It can be seen from the experimental investigations of flow over triangular wings [5, 7, 8] that the flow on the upper wing surface is greatly influenced by the flow conditions at the edges, which depend on $\alpha_{n}$ and $M_{n}$ in the section normal to the edge, and on the shape of the edge. Numerical calculations using the Navier-Stokes equations of supersonic flow over a slender triangular wing [9] at angle of attack also show a strong influence of $M_{n}$ on the pattern of separated flow on its upper surface. A detailed classification of patterns of separated flow over wings was given in [8].

The results of the papers listed here, and of a number of other papers, give both a qualitative and a quantitative idea of the structure of flow and heat transfer on the upper surface of a triangular wing. Nevertheless, a whole series of special features of this flow remain unknown. For instance, there are no criteria for the existence of a particular type of separated flow and of the influence of the flow pattern on the heat transfer. There has been no detailed investigation of the influence of Mach and Reynolds numbers, the shape of the wing surface, and the superstructure on its leeward side on the aerodynamic wing heating. Some of these questions are examined in the present paper.

1. The experiments were conducted at $M_{\infty}=3,4$, and 5 on models of a triangular wing with leading edge sweepback angle of $x=75^{\circ}$ (Fig. 1). The semivertex angle of the sharp leading edge of models 1 and 4 was $\delta_{n}=29.5^{\circ}$ in the normal cross section. The upper surface of model 4 carried a sharp half cone of semivertex angle $5^{\circ}$. Model 2 is a sharp half cone with semivertex angle $15^{\circ}$, and model 3 is a planar wing with leading edge blunting radius


Fig. 1

[^0]$r=3 \mathrm{~mm}$. The length of the models is $L=210-250 \mathrm{~mm}$. The models were made of fiberglas material AG-4, since the heat transfer was measured with the method of a thermally sensitive coating [10]. Visualization of flow over the models was accomplished by the laser Schlieren method [11] and by erosion of droplets of visualizing material [12]. The Reynolds number computed from the parameters of the unperturbed incident stream and the model length $L$ was $\operatorname{Re}_{\mathrm{L}}=(2.5-6.9) \cdot 10^{6}$ for $\alpha=0-15^{\circ}$. The shock wave was always detached from the model leading edges.

Figure 2 shows photographs of the upper surface of a triangular wing (model 1), where one can see a picture of melting of the thermal indicator on a background of limiting streamlines made during the experiment with $M_{\infty}=4$, $\operatorname{Re}_{L}=3 \cdot 10^{6}$, and $\alpha=5^{\circ}$. The dark regions on the wing surface are regions of increased values of heat flux. The boundaries of these regions correspond to $S t_{\infty}=1.1 \cdot 10^{-3}(\mathrm{a})$ and $S t_{\infty}=0.7 \cdot 10^{-3}(\mathrm{~b})$. Inside the boundaries the values of $S t_{\infty}$ are larger, and outside the boundaries (in the white regions) they are less than on the boundary; $S t_{\infty}=q_{w} /\left(\rho_{\infty} u_{\infty} c_{p}\left(T_{0}-T_{w}\right)\right)\left(q_{w}\right.$ is the local heat flux $)$; $\rho_{\infty}$ and $u_{\infty}$ are the density and velocity of the unperturbed stream; $c_{p}$ is the specific heat of air; $T_{0}$ is the stagnation temperature of the incident flow; and $T_{w}$ is the model wall temperature). In computing $\mathrm{q}_{\mathrm{w}}$ we used the stagnation temperature $\mathrm{T}_{0}$ as a determinant.

The narrow zones of increased values of $S t_{\infty}$ in the middle wing areas located along rays which are symmetrical relative to the angle-of-attack plane are associated with reattachment of the flow that separated from the wing leading edges. The region of strong heat transfer in the vicinity of the plane of symmetry at the rear part of the model is linked to transition of the laminar flow to turbulent. The rays of maximum values of $S t_{*}$ emanate, as was true in [2, 3], from a point located on the wing axis of symmetry at a certain distance from its apex. It was stated in $[2,3]$ that the lines of $\mathrm{St}_{*}$ are displaced from the outflow line in the region of the $S$-shaped stream lines. In fact, the $S t$. lines coincide with the outflow lines 1 (Fig. 2) on which there is reattachment of the flow that separated from the leading edges (the discharge line 3 ).

In the vicinity of the plane of symmetry and the line of repeated separation 2 , minimal values of laminar heat transfer are observed (Figs. 2 and 3). For $M_{\infty}=3, \operatorname{Re} L=3.2 \cdot 10^{6}$,


Fig. 2


Fig. 3
and $\alpha=5.5^{\circ}$, the $S t_{*}$ lines emanate from the wing vertex (Fig. 3a), and for $M_{\infty}=5, \mathrm{Re}_{\mathrm{L}}=$ $2.6 \cdot 10^{6}$ they radiate at a distance $x \approx 25 \mathrm{~mm}$ from the vertex (Fig. 2b), i.e., with increase of $M_{\infty}$, the point of divergence of the reattachment lines moves further from the wing vertex. Naturally, the origin of the lines of the second separation 2 also move away from the vertex, and near this point there arises a cirrus-like structure of limiting stream lines, which was noted earlier [2, 3].

In all the cases investigated, the primary separation on model 1 occurs at the leading edges, and only for $M_{\infty}=5$ and $0<\alpha \leq 5.5^{\circ}$ on the upper wing surface at some distance from the leading edges. For $\alpha=5.5^{\circ}$, the separation line 3 is still on the upper wing surface immediately behind the leading edges (Fig. 3b).

Figure 3 also shows photographs of the flow visualization above the model surface by the laser Schlieren method [11] in a section at distance $\mathrm{x}=200 \mathrm{~mm}$ from the wing vertex, normal to the incident stream. The photograph was taken above the trailing wing edge at an angle of about $25^{\circ}$ relative to the upper surface. For $M_{n}<1$ (Fig. 3a, $M_{n}=0.82$ ) the flow is subsonic with two vortices above the wing and a secondary separation (scheme 3 in [8]), and for $M_{n}>1$ (Fig. 3b, $M_{n}=1.36$ ) the vortices transfer to the separated regions adjacent to the wing surface, which move close to the symmetry plane (scheme 5 in [8]).
2. The change in the flow conditions over the wing leads to a restructuring of the flow scheme on its leeward surface and, therefore, to a change of the magnitude and position of the heat-flux peaks. As was true also in [3], the wing angle of attack is the governing influence in this sense. For $M_{\infty}=5$, it is the same in the present study as on the wing with $\mathrm{X}=75^{\circ}$ in [3]. Two peaks of $\mathrm{St}_{\infty}$ in the wing cross section.(model 1) reach the maximum value for $\alpha \simeq 2-5.5^{\circ}$. With further increase of $\alpha$ the peaks decrease sharply, and for $\alpha>$ $10^{\circ}$ a maximum appears in the symmetry plane. With increase of $\alpha$ the peaks move close to the symmetry plane.

For $M_{\infty}=3$ and 4, the influence of $\alpha$ on the $S t_{\infty}$ distribution in the cross section of model 1 , corresponding to $\mathrm{Re}_{\mathrm{x}}=\operatorname{Re}_{1} \mathrm{x}=1.2 \cdot 10^{6}$ (Fig. 4a) is qualitatively similar to what is observed for $M_{\infty}=5$.

Points $1-5$ of Fig. 4 correspond to $\alpha=0,3,5,10$, and $15^{\circ}$. Here and below, because of symmetry of the heat-transfer distribution, data are presented on the graphs for one half of the section. Since at this $M_{\infty}$ the flow over the wing separates from the leading edges, starting at $\alpha=0\left(M_{n}<1\right)$, there are local maximum heat-flux values even at zero angle of attack. The largest values of $\mathrm{St}_{\infty}$ in the peaks were measured for $\alpha \simeq 0-5^{\circ}$. The influence of $\alpha$ on $S t_{*}$ is more pronounced than at $M_{\infty}=5$.

The distribution of $S t_{\infty}$ for $M_{\infty}=5$ in the section corresponding to $R e_{x}=1.4 \cdot 10^{6}$ on the leeward surface of a semi-cone (model 2) and of a wing with a semi-cone (model 4) is


Fig. 5
shown in Fig. 4b. The heat-flux peak for $\alpha=0$ on mode1 4 is associated with interference of the boundary layer on the wing with a shock wave induced by the semi-cone. On model 2, the semi-cone for $\alpha=0$ separated flow occurs on the leeward surface, associated with this local maximum of heat transfer.
3. For $M_{\infty}=5$ and $\alpha=5^{\circ}$, a change in shape of the wing cross section does not change the basic nature of the heat-flux distribution on its upper surface (Fig. 5a). On the models of the wing 1 , the semi-cone 2, and the wing plus semi-cone 4 , the location of the peak of $S t_{\infty}$, associated with reattachment of the flow that separated in the vicinity of the leading edges, does not change ( $z / \mathrm{x} *-0.07$ ). For model 4 it is located on the superstructure and is then a minimum. The second heat-flux peak on this model (at $z / x \approx-0.11$ ) is due to the repeated reattachment on the wing of the boundary layer that separated at the superstructure. On the planar wing with the cylindrical edges (model 3), the separation line is displaced to the leeward surface [7]. The reattachment line (the St. line), and, therefore, the heatflux peak on this wing, moves close to the wing symmetry plane. For the wing cross section examined, the largest values of $\mathrm{St}_{\infty}$ in the heat-transfer peaks on its upper surface at the section corresponding to $\mathrm{Re}_{\mathrm{x}} \simeq 1.3 \cdot 10^{6}$ were obtained on the wing with sharp leading edges (model 1, $\mathrm{St}_{*} \approx 3 \cdot 10^{-3}$ ), and the minimal values were obtained on model 4 ( $\mathrm{St}_{*} \approx 1.3 \cdot 10^{-3}$ ).

On the wing with sharp edges (Fig. 5a), with increase of $M_{\infty}$ from 3 to 5 (points 1-3 correspond to $M_{\infty}=3,4,5$ ) the angle of divergence of the $\mathrm{St}_{*}$ lines decreases, and the heattransfer peak becomes less pronounced. The value of $\mathrm{St}_{*}$ at the peak increases here by approximately a factor of 2 . The shaded points show the distribution of $\mathrm{St}_{\infty}$ at the same section of the planar surface of the wing for the case when it is windward with $M_{\infty}=5$ and $\alpha=$ $5^{\circ}$. In the middle part of the wing there is transition from laminar to turbulent boundary layer. For $M_{\infty}=5$ the peak values of $\mathrm{St}_{\infty}$ on the leeward wing surface exceed the turbulent heat-transfer level on the windward surface by approximately a factor of 2 . The repeated reattachment of the boundary layer separated at the separation line 2 (see Figs. 2 and 3) does not cause a sharp increase of heat transfer (Fig. 5a).

The installation of a turbulence generator near the vertex on the leeward surface of model 1 (see Fig. 1) in the form of a band of sandpaper roughness with the characteristic size of roughness element of $0.3-0.5 \mathrm{~mm}$ at $\mathrm{M}_{\infty}=5$ and $\alpha=5.5^{\circ}$ does not introduce appreciable changes in the transverse distribution of $\mathrm{St}_{\infty}$ (Fig. 5b on the left, where the shaded points


Fig. 6
are data obtained with the turbulence generator, and the open points are data without the turbulence generator). The increase of $R e_{x}$ (unit Reynolds number) for $M_{\infty}=3$ has also practically no influence on the heat-flux distribution and the values of $S t_{*}$ at a fixed transverse section ( $x / L=0.36$ ) of the upper surface of model 1 ( $F i g$. $5 b$, right side, where points $1-3$ correspond to $\operatorname{Re}_{x} \cdot 10^{6}=1.4,1.7,2.3$. An exception is the region in the vicinity of the plane of symmetry, where there is transition of laminar flow to turbulent as Re increases. The graph shows calculated values of $\mathrm{St}_{\infty}$ for laminar [13] and turbulent boundary layers [14] on a flat plate set at zero angle of attack. The maximum values of $\mathrm{St}_{\infty}$ at the peaks for $M_{\infty}=3$ correspond approximately to the turbulent heat-flux level on the flat plate.
4. The results of this study indicate that, with increase of $M_{\infty}$ on the wing with $X=$ $75^{\circ}$ the peaks of $S t_{\infty}$ move close to the wing plane of symmetry and become less pronounced. Here, on the upper surface of the wing, there are developed regions of separation (see Fig. 3). Figure 6a for $\alpha=5^{\circ}$ compares the present results and those of [6] ( $x=80^{\circ}, M_{\infty}=10.3$, $R e_{1}=6 \cdot 10^{6} 1 / \mathrm{m}$ ) at the cross section corresponding to $R e_{X} \simeq 1.3 \cdot 10^{6}$, where $S t_{0}$ is the Stanton number corresponding to its value on the axis of symmetry. The experimental points 1 and 2 were obtained in the present study for $M_{\infty}=3$ and 5 , respectively, and points 3 were taken from [6]. The absence of peaks on the upper wing surface in [6] is due to the fact that, for these flow conditions, there is apparently only incipient separation on the leeward side.

As was noted above, the angle $\beta$ of divergence of the $\mathrm{St}_{*}$ lines (the reattachment line) on the leeward wing surface depends on $M_{\infty}$ and $\alpha$. It is shown in Fig. 6b that the values of the product $M_{\infty}$ tan $\beta$ as a function of $\alpha$ for the wing with $\chi=75^{\circ}$ and sharp leading edges at $M_{\infty}=3-5$ are averaged by a single curve. Points $1-3$ correspond to $M_{\infty}=3$, 4, 5, and the shaded points were obtained on the semi-cone. The data for the semi-cone for $M_{\infty}=5$ are also described by this curve.

This behavior of the separation and the heat-flux peaks is explained by the elementary flow scheme in the wing cross section, by analogy with transverse flow over a circular cylinder with two symmetric vortices on its leeward side. The circulation of the vortices is determined by the Zhukovskii condition (zero velocity at the plate edges, here the wing, which maps conformally into a cylinder). The transverse velocity is

$$
v=\frac{4 u a^{2} \sin 2 \vartheta[(a+1 / a) \cos \varphi-\cos \vartheta]}{\left[1+a^{2}-2 a \cos (\vartheta-\varphi)\right]\left[1+a^{2}-2 a \cos (\vartheta+\varphi)\right]} .
$$

Here $u=u_{\infty} \sin \alpha ; a$ and $\varphi$ are the cylindrical coordinates of the vortex; the angle $\theta$ is associated with the coordinate of a point on the plate $\eta$; $\sin \vartheta=2 \eta(-2 \leq \eta \leq 2)$. The velocity $v$ goes to zero on the plate ( $\eta>0$ ), only if the vortex is located in a very small region near the wing leading edge, i.e., if the angles of attack are small. Then $v=0$ corresponds to the line of divergence. Here the separation regions are small and are located near the surface.

At large $\alpha$, when the vortices leave these regions, the velocity goes to zero for $\eta=$ 0 , i.e., in the middle of the wing. Since any contour of cross section maps to a circle, the above is extended to the flat plate, the semi-cone, and to the wing with a semi-cone.

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